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Linearization and symmetry breaking in nonlinear SU(3) × SU(3)

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Abstract. A method for constructing linear representations from nonlinearly transforming pseudoscalar mesons is developed for the group SU(3) × SU(3). The use of these functions as symmetry breaking terms is made apparent in phenomenological Lagrangians and current theories where the Schwinger term plays a particularly natural role as the symmetry breaker.

The general theory of nonlinear realizations of chiral groups is well established (Coleman *et al* 1969, Isham 1969). For the group SU(2) × SU(2), the algebraic approach of Weinberg (1968) and the matrix method associated with the name of Gürsey (Chang and Gürsey 1967) both lead to general closed form expressions for the transformation laws. Although in principle, both these methods are capable of generalization to the group SU(3) × SU(3) (Macfarlane *et al* 1970), it is only recently that the Gürsey approach has been reformulated in such a way that completely general results can be obtained (Barnes *et al* 1972a, b).

For the nonlinearly transforming pseudoscalar mesons, an invariant Lagrangian can be constructed which displays the Goldstone nature of the mesons in the symmetric theory. In order to form a more realistic model, symmetry breaking terms must be added to the invariant Lagrangian. As Weinberg (1968) has pointed out, a natural way to describe the symmetry breaking part of the Lagrangian is by its transformation properties under the action of the chiral group, thus retaining the coordinate independence of the Lagrangian. Then, for SU(2) × SU(2), an isoscalar symmetry breaking term can be chosen as the isoscalar from the $(N/2, N/2)$ representation, where $N/2$ is the SU(2) spin label of the representation. When $N = 1$, the pion scattering lengths from the model are equivalent to those calculated by Weinberg using current algebra techniques (Weinberg 1966), and which now have some measure of experimental confirmation.

At the SU(3) × SU(3) level, the strong interaction Lagrangian breaks both the full chiral symmetry and the SU(3) symmetry generated by the vector charges but retains isospin invariance. To ensure that the Gell-Mann–Okubo mass formula is satisfied, we choose the even parity SU(3) singlet and eighth component of the even parity SU(3) octet representation from the $(m, \bar{m}) + (\bar{m}, m)$ representation of the full chiral group, where m labels the dimension of the representation. Although any of these representations can be used, two in particular deserve more consideration. When $m = 3$, we have the model of Gell-Mann *et al* (1968) which has the unique property of SU(2) × SU(2) symmetry as the pion mass tends to zero, and which also contains the Weinberg scattering lengths for pions. For $m = 8$, the model of interest is that studied by Barnes and Isham (1970a, b). They show that a current–current theory of the Sugawara type (Sugawara 1968) can be developed from an underlying nonlinear Lagrangian and that the q number Schwinger term that appears in the commutation relations is a natural candidate for symmetry breaking in a theory of this kind. The Schwinger term transforms as an (8, 8) representation of $SU(3)\times SU(3)$, so that by choosing the singlet and eighth component of the symmetric octet they have the desired SU(3) properties. Their model obviously cannot coincide with the $(3,\bar{3})+(\bar{3},3)$ model and is thus less favourable on present experimental knowledge.

We want to show in this paper that it is possible to construct all the functions of the nonlinearly transforming meson fields that form a linear $(m,\bar{m})$ representation. We can then, as in the $SU(2)\times SU(2)$ case, form the symmetry breaking Lagrangian in any coordinate frame, to any order. Finally, the $(3,\bar{3})+(\bar{3},3)$ model can be reconciled with the current–current model, whose Schwinger term transforms as an (8, 8) representation, by considering the nonlinear constraints that the Schwinger term has to obey. Thus we demonstrate that it is still possible for the Schwinger term, that appears naturally in the current commutators, to play a role in symmetry breaking even when we require it to be from the $(3,\bar{3})+(\bar{3},3)$ representation.

By way of an introduction, we study the equivalent problem for the $SU(2)\times SU(2)$ group. No new results are obtained, but we feel that the simplicity of the algebra in this case allows the reader who is unfamiliar with the methods that we intend to use, to gain some insight into the ease with which they may be applied. Now, the $(N/2,N/2)$ representation of $SU(2)\times SU(2)$ has two Casimir invariants. Further, if we choose the isoscalar component, we then have an object that satisfies

$$[Q_i^{\mathrm{V}}, S^N] = 0 \qquad i = 1,2,3 \tag{1}$$

$$[Q_i^{\mathrm{L}}, [Q_i^{\mathrm{L}}, S^N]] = \tfrac{1}{4}N(N+2)S^N \qquad \begin{pmatrix}\text{summation convention}\\ \text{on repeated indices}\end{pmatrix} \tag{2}$$

$$[Q_i^{\mathrm{R}}, [Q_i^{\mathrm{R}}, S^N]] = \tfrac{1}{4}N(N+2)S^N, \tag{3}$$

where S^N is the isoscalar in the $(N/2,N/2)$ representation. Q_i^{L} and Q_i^{R} are one half the sum and one half the difference of the physical vector and axial vector generators, and generate the commuting SU(2) subgroups. Using this information we obtain

$$[Q_i^{\mathrm{A}}, [Q_i^{\mathrm{A}}, S^N]] = N(N+2)S^N \tag{4}$$

as an equation on S^N, and since π^2 is the only isoscalar available in the theory, we may consider equation (4) as a differential equation in the variable π^2. We see however, from a glance at any of the works on nonlinear realizations given in the references, that π^2 does not have a simple axial transformation and instead we follow the more fundamental approach developed in Barnes *et al* (1972a, b), which the reader should consult for further information. We can write a unitary unimodular matrix as

$$U = \mathrm{e}^{\mathrm{i}\theta}N^+ + \mathrm{e}^{-\mathrm{i}\theta}N^- \tag{5}$$

where θ is a function of π^2 and $N^\pm = \tfrac{1}{2}(1\pm N_i\tau_i)$ are projection operators with $N_i^2 = 1$ and τ_i the Pauli matrices. The physical pion field is related to N_i by $\pi_i = (\pi^2)^{1/2}N_i$ so that U is a matrix function of the pions only. The transformation properties of U under the chiral group elements can be described by

$$[Q_i^{\mathrm{A}}, U] = \tfrac{1}{2}\{U, \tau_i\} \tag{6}$$

which implies that

$$[Q_i^{\mathrm{A}}, \theta] = -\mathrm{i}N_i \tag{7}$$

and

$$[Q_i^{\mathrm{A}}, N_j] = -\mathrm{i}\cot\theta(\delta_{ij}-N_iN_j) \tag{8}$$

as demonstrated in Barnes *et al* (1972b). We have obtained by this parametrization of U, an isoscalar which has simple coordinate independent transformation properties so that using

$$[Q_i^{\mathrm{A}}, S^N] = \frac{\mathrm{d}S^N}{\mathrm{d}\theta}[Q_i^{\mathrm{A}}, \theta] \tag{9}$$

in equation (4) we obtain the differential equation

$$\frac{\mathrm{d}^2S^N}{\mathrm{d}\theta^2}+2\cot\theta\frac{\mathrm{d}S^N}{\mathrm{d}\theta}+N(N+2)S^N = 0. \tag{10}$$

This is Chebyshev's differential equation and has solutions, finite for $\theta = 0$, which are

$$S^N = \frac{\sin(N+1)\theta}{\sin\theta}, \tag{11}$$

as can easily be seen by writing the differential equation for the function $S^N\sin\theta$. A similar re-arrangement when dealing with the SU(3) × SU(3) problem also leads to a simple solution. By considering various multiple commutators of S^N with the group generators, and using the known infinitesimal transformation laws of θ and N_i, all the components of the linear representation can be found. This completes our introduction to the problem for the SU(2) × SU(2) case, however, let us add the comment that once the equation to be satisfied by the isoscalar from the $(N/2, N/2)$ representation, equation (4), has been derived, it is then just a matter of finding a suitable function of π^2 to solve the differential equation. Another example of a different parametrization can be found in Barnes and Dondi (1971) where the basic isoscalar is taken to be the isoscalar of the $(\frac{1}{2}, \frac{1}{2})$ representation. This reference also contains a full discussion of the connection between nonlinear realizations and Schwinger terms for an SU(2) × SU(2) current model and we will not go into further details of this aspect of our problem here.

In extending the method described above to the group SU(3) × SU(3), we have to remember that there are two Casimir invariants for each commuting SU(3) subgroup, and that as there is a symmetric, invariant, third rank tensor, we can construct two scalars under the vector SU(3) subgroup out of the single octet of pseudoscalar mesons.

Now, the vector SU(3) scalar from the $(m, \bar{m})$ representation, $S^{m,\bar{m}}$, satisfies

$$[Q_i^{\mathrm{V}}, S^{m,\bar{m}}] = 0 \qquad i = 1, 2\ldots 8 \tag{12}$$

$$[Q_i^{\mathrm{L}}, [Q_i^{\mathrm{L}}, S^{m,\bar{m}}]] = C_2^m S^{m,\bar{m}} \tag{13}$$

$$d_{ijk}[Q_i^{\mathrm{L}}, [Q_j^{\mathrm{L}}, [Q_k^{\mathrm{L}}, S^{m,\bar{m}}]]] = C_3^m S^{m,\bar{m}} \tag{14}$$

$$[Q_i^{\mathrm{R}}, [Q_i^{\mathrm{R}}, S^{m,\bar{m}}]] = C_2^{\bar{m}} S^{m,\bar{m}} \tag{15}$$

$$d_{ijk}[Q_i^{\mathrm{R}}, [Q_j^{\mathrm{R}}, [Q_k^{\mathrm{R}}, S^{m,\bar{m}}]]] = C_3^{\bar{m}} S^{m,\bar{m}}, \tag{16}$$

where C_2^m is the quadratic Casimir eigenvalue for the representation labelled by m, and

C_3^m is the cubic Casimir eigenvalue in the same representation. In terms of the usual (p, q) labelling of SU(3) states, the dimension is given by

$$m = \tfrac{1}{2}(p+1)(q+1)(p+q+2), \tag{17}$$

and the Casimirs take the values

$$C_2^m = \tfrac{1}{3}(p^2+pq+q^2)+(p+q) \tag{18}$$

$$C_3^m = \tfrac{1}{18}(p-q)(2p+q+3)(p+2q+3). \tag{19}$$

Since $C_2^m = C_2^{\bar{m}}$ and $C_3^m = -C_3^{\bar{m}}$, we can combine equations (12)–(16) to give

$$[Q_i^{\mathrm{A}}, [Q_i^{\mathrm{A}}, S^{m,\bar{m}}]] = 4C_2^m S^{m,\bar{m}} \tag{20}$$

and

$$d_{ijk}[Q_i^{\mathrm{A}}, [Q_j^{\mathrm{A}}, [Q_k^{\mathrm{A}}, S^{m,\bar{m}}]]] = 8C_3^m S^{m,\bar{m}} \tag{21}$$

which play an equivalent role in SU(3) × SU(3) to that of equation (4) in SU(2) × SU(2), and are to be considered as differential equations in terms of the SU(3) scalars of the theory. The commutators of the axial generators with both $M_i M_i$ and $d_{ijk} M_i M_j M_k$, the SU(3) scalars formed from the pseudoscalar meson octet M_i, and the totally symmetric tensor d_{ijk}, are complicated objects to deal with and we again rely on the more basic approach developed in Barnes *et al* (1972a, b) to solve the problem.

The SU(3) unitary, unimodular matrix can be written as

$$U = U_0 P_0 + U_+ P_+ + U_- P_- \equiv U_\alpha P_\alpha \tag{22}$$

where P_α are the projection operators of Barnes *et al* (1972a) (we advise the reader to consult this reference for full details as they are too important to review briefly in this paper), and U_α each have modulus unity and multiply together to give unity. Furthermore, U_α are functions of the two SU(3) scalars, and the projection operators are known explicitly in terms of the meson fields (Barnes *et al* 1972a). We can use particular exponential expressions for U_α, and write

$$U = \mathrm{e}^{\mathrm{i}2R} P_0 + \mathrm{e}^{-\mathrm{i}(R+\mathscr{I})} P_+ + \mathrm{e}^{-\mathrm{i}(R-\mathscr{I})} P_- \tag{23}$$

where R and $\mathscr{I}$ are related to the θ of Barnes *et al* (1972a) by $\theta = R + \mathrm{i}\mathscr{I}/\sqrt{3}$.

The axial transformation of U is given by

$$[Q_i^{\mathrm{A}}, U] = \tfrac{1}{2}\{U, \lambda_i\}, \tag{24}$$

where λ_i are the Gell-Mann matrices, which implies in turn that

$$[Q_i^{\mathrm{A}}, \mathscr{I}] = -\mathrm{i}s_i \tag{25}$$

and

$$[Q_i^{\mathrm{A}}, R] = \mathrm{i}q_i/\sqrt{3} \tag{26}$$

where s_i and q_i are the special and charge vectors of Barnes *et al* (1972a). They will not appear in our final equation for $S^{m,\bar{m}}$, however, as in the SU(2) × SU(2) problem, their commutation relations with the axial generators are required, and these are given by

$$[Q_i^{\mathrm{A}}, q_j] = \mathrm{i}\tfrac{1}{2}\sqrt{3}(\cot(+)P_{ij}^{(+)} + \cot(-)P_{ij}^{(-)}) \tag{27}$$

$$[Q_i^{\mathrm{A}}, s_j] = -\mathrm{i}(\cot \mathscr{I} P_{ij}^{(0)} + \tfrac{1}{2}\cot(+)P_{ij}^{(+)} - \tfrac{1}{2}\cot(-)P_{ij}^{(-)}) \tag{28}$$

where $(+) = (3R+\mathscr{I})/2$, $(-) = (3R-\mathscr{I})/2$ and the tensors are given as functions of q_i and s_i as (Barnes 1972)

$$P_{ij}^{(0)} = \tfrac{1}{3}(\delta_{ij}+2\sqrt{3}\,d_{ijk}q_k+q_iq_j-3s_is_j) \tag{29}$$

$$P_{ij}^{(\pm)} = \tfrac{1}{3}[\delta_{ij}-\sqrt{3}\,d_{ijk}q_k-2q_iq_j\mp\{3d_{ijk}s_k-\sqrt{3}(q_is_j+s_iq_j)\}]. \tag{30}$$

The simplicity and coordinate independent nature of the commutators given in equations (25) and (26) leads us to choose R and $\mathscr{I}$ as our basic independent SU(3) scalars with which to set up the differential equations for the function $S^{m,\bar{m}}$. Using

$$[Q_i^{\mathrm{A}}, S^{m,\bar{m}}] = \frac{\partial S^{m,\bar{m}}}{\partial R}[Q_i^{\mathrm{A}}, R]+\frac{\partial S^{m,\bar{m}}}{\partial \mathscr{I}}[Q_i^{\mathrm{A}}, \mathscr{I}] \tag{31}$$

together with the commutators given in equations (25)–(28) and the properties of the charge and special vectors, straightforward manipulation gives

$$\frac{1}{3}\frac{\partial^2 S^{m,\bar{m}}}{\partial R^2}+\frac{\partial^2 S^{m,\bar{m}}}{\partial \mathscr{I}^2}+\frac{\partial S^{m,\bar{m}}}{\partial R}(\cot(+)+\cot(-))$$
$$+\frac{\partial S^{m,\bar{m}}}{\partial \mathscr{I}}(2\cot\mathscr{I}+\cot(+)-\cot(-))+4C_2^mS^{m,\bar{m}} = 0 \tag{32}$$

as the differential equation that arises from equation (20). Although this is a partial differential equation, it obviously bears a marked resemblance to its SU(2) counterpart. Equation (21) leads, in a similar way, to a third order partial differential equation, which we leave the reader to derive if he wishes. Transforming equation (32) into a differential equation for the function $F = \sin\mathscr{I}\sin(+)\sin(-)S^{m,\bar{m}}$ we obtain the simple form

$$\frac{1}{3}\frac{\partial^2 F}{\partial R^2}+\frac{\partial^2 F}{\partial \mathscr{I}^2} = -4(C_2^m+1)F \tag{33}$$

whilst the third order equation for F from equation (21) is

$$\frac{1}{9}\frac{\partial^3 F}{\partial R^3}-\frac{\partial^3 F}{\partial R\partial^2\mathscr{I}} = -\mathrm{i}8C_3^mF. \tag{34}$$

We may solve equation (33) by assuming a separable solution $F = F_1(R)F_2(\mathscr{I})$, then

$$F_1 = A\,\mathrm{e}^{\sqrt{(3\alpha)}R}+B\,\mathrm{e}^{-\sqrt{(3\alpha)}R} \tag{35}$$

and

$$F_2 = C\,\mathrm{e}^{\mathrm{i}k\mathscr{I}}+D\,\mathrm{e}^{-\mathrm{i}k\mathscr{I}} \tag{36}$$

where $k = \{4(C_2^m+1)+\alpha\}$, and A, B, C, D, and α are arbitrary constants. Using the separation in the second equation on F, we obtain the first order equation

$$\frac{\partial F_1}{\partial R} = \frac{-\mathrm{i}(2C_3^m)}{\frac{1}{3}\alpha+(C_2^m+1)}F_1 \tag{37}$$

which implies

$$3\alpha\{\tfrac{1}{3}\alpha+(C_2^m+1)\}^2 = -(2C_3^m)^2, \tag{38}$$

and that either A or B is zero. We may arbitrarily choose $B = 0$ without loss of generality. The cubic equation in α has roots $-(p-q)^2/3$, $-(p+2q+3)^2/3$ and $-(2p+q+3)^2/3$ as can easily be seen by using the expressions for the Casimir eigenvalues in terms of p and q. The corresponding values of k are $(p+q+2)$, $(p+1)$, and $(q+1)$ respectively. We thus have as the general solution of the differential equations (33) and (34) which is

$$\begin{aligned} F = A_1\, e^{-i(p-q)R}(e^{i(p+q+2)\mathscr{I}} + B_1\, e^{-i(p+q+2)\mathscr{I}}) \\ + A_2\, e^{-i(p+2q+3)R}(e^{i(p+1)\mathscr{I}} + B_2\, e^{-i(p+1)\mathscr{I}}) \\ + A_3\, e^{i(2p+q+3)R}(e^{i(q+1)\mathscr{I}} + B_3\, e^{-i(q+1)\mathscr{I}}) \end{aligned} \tag{39}$$

where A_i and B_i, $i = 1, 2, 3$ are arbitrary constants to be determined by the boundary condition that the SU(3) scalar $S^{m,\bar{m}}$ is finite even when the angles $\mathscr{I}$, $(+)$, $(-)$ are zero. This implies, by the connection between F and $S^{m,\bar{m}}$, that F is zero at these points and therefore that $B_1 = B_2 = B_3 = -1$ and $A_2 = A_3 = -A_1$ so that our final answer is that the function of the meson fields transforming as a scalar under the vector SU(3) subgroup, and belonging to the $(m, \bar{m})$ representation of the full chiral SU(3) × SU(3) is, up to a multiplicative arbitrary constant,

$$\begin{aligned} S^{m,\bar{m}} = \{e^{-i(p-q)R} \sin(p+q+2)\mathscr{I} + e^{-i(p+2q+3)R} \sin(p+1)\mathscr{I} \\ + e^{i(2p+q+3)R} \sin(q+1)\mathscr{I}\}/\sin\mathscr{I}(\cos\mathscr{I} - \cos 3R). \end{aligned} \tag{40}$$

It is interesting to note, that for the selfconjugate representations which have $p = q$, this solution reduces to

$$\frac{\sin(p+1)\mathscr{I}\,\sin(p+1)(+)\,\sin(p+1)(-)}{\sin\mathscr{I}\,\sin(+)\,\sin(-)} \tag{41}$$

which further demonstrates the similarity of the methods of dealing with the SU(2) × SU(2) and SU(3) × SU(3) case.

The solution that we have obtained for $S^{m,\bar{m}}$ together with the commutators in equations (25)–(28) allows us to generate all of the components of the linear representations of the $(m, \bar{m})$ type as functions of the meson fields. As an obvious example, the $(3, \bar{3})$ representation has nine components, and the octet is produced from the scalar by forming the commutator of $S^{3,\bar{3}}$ with an axial generator

$$[Q_i^{A}, S^{3,\bar{3}}] = O_i^{3,\bar{3}}. \tag{42}$$

The (8,8) tensor has among its SU(3) irreducible parts, two octets, and noticing that the scalar has positive parity, we can define

$$[Q_i^{A}, S^{8,8}] = iO_{Ai}^{8,8} \tag{43}$$

and

$$d_{ijk}[Q_j^{A}, [Q_k^{A}, S^{8,8}]] = O_{Si}^{8,8} \tag{44}$$

which are the antisymmetric and symmetric octets respectively. Further manipulation of multiple commutators gives the other SU(3) irreducible representations in (8, 8).

The physical significance of this calculation is that we are now able to produce phenomenological Lagrangians with manifest coordinate invariance. The $(3, \bar{3}) + (\bar{3}, 3)$ model has a symmetry breaking term

$$C_1(S^{3,\bar{3}} + S^{\bar{3},3}) + C_2(O_8^{3,\bar{3}} - O_8^{\bar{3},3}) \tag{45}$$

where C_i are arbitrary constants, and in terms of the angles R and $\mathscr{I}$ and the vectors s_i and q_i this is just

$$C_1 2(\cos 2R + 2\cos \mathscr{I} \cos R) + C_2 4\left((\cos R \cos \mathscr{I} - 2\cos 2R)\frac{q_8}{\sqrt{3}} + \sin R \sin \mathscr{I} s_8\right) \tag{46}$$

whilst the (8, 8) model contributes terms

$$C_1' S^{8,8} + C_2' O^{8,8}_{S8} \tag{47}$$

which can also be written as a function of R, $\mathscr{I}$, q_i and s_i. In fact, it is possible to construct symmetry breaking scalar and octet pieces for any $(m, \overline{m}) + (\overline{m}, m)$ representation which are given explicitly as functions of the meson fields.

Now let us turn our attention to the current–current model of Barnes and Isham (1970a, b). Their symmetry breaking term in exponential coordinates is exactly equivalent to that given in equation (47), as we shall later demonstrate. Further, we wish to show that the Schwinger term can be used to describe any mode of symmetry breaking and thus their model need not be discarded simply on the grounds of having the wrong pion scattering lengths. By comparing the form of the operator Schwinger term in Barnes and Isham (1970a) with the work of Barnes *et al* (1972a), we can easily observe that it may be written as a function of the matrix U:

$$S^{\mathrm{LR}}_{ij} = -\frac{f_\pi^2}{8} \mathrm{Tr}(U\lambda_i U^+ \lambda_j) \tag{48}$$

where f_π is the pion decay constant, and for exponential coordinates

$$U = \exp(\mathrm{i} M_p \lambda_p / f_\pi), \tag{49}$$

we reproduce the special case considered by Barnes and Isham, that is

$$S^{\mathrm{LR}}_{ij} = -\frac{f_\pi^2}{4} \exp(2f_\pi^{-1} M_k f_{kij}) \tag{50}$$

where f_{kij} are the usual Gell-Mann structure constants (Gell-Mann and Ne'eman 1964). We may also use equation (48) to confirm our earlier results, since the trace of S^{LR}_{ij}, and $\mathrm{d}_{ijk} S^{\mathrm{LR}}_{jk}$ form scalar and octet components of an (8, 8) tensor.

Note that this Schwinger term is more restricted than the one considered in Barnes and Isham (1970b), since not only does it satisfy

$$S^{\mathrm{LR}}_{ij} S^{\mathrm{LR}}_{kj} = \frac{f_\pi^4}{16} \delta_{ik}, \tag{51}$$

as the constraint implied by the Schwinger condition and the construction of the energy momentum tensor to avoid parity doubling, but it also obeys (Macfarlane 1968)

$$(-4f_\pi^{-2})^3 d_{abc} S^{\mathrm{LR}}_{ia} S^{\mathrm{LR}}_{jb} S^{\mathrm{LR}}_{kc} = d_{ijk}. \tag{52}$$

For Sugawara type theories (Sugawara 1968), this condition requires

$$d_{ijk}(R_i^\mu R_j^\nu R_k^\rho + L_i^\mu L_j^\nu L_k^\rho) = 0 \tag{53}$$

where R_a^μ and L_a^μ are the right and left currents respectively, a is the SU(3) index and μ the Lorentz index. This is rather an exotic condition, and has played no part in the development of current theories so far. However, from our point of view, it is just the two nonlinear constraints of equations (51) and (52) that reduce the number of independent components in S_{ij}^{LR} from sixty four to eight and allow it to be written as a function of the U matrix. It is possible (Macfarlane 1968) to invert the expression for S_{ij}^{LR} as a function of U, and obtain

$$U = \frac{\frac{1}{3}(1+\mathrm{Tr}\,R)+\frac{1}{2}\lambda_i K_i}{\{\frac{1}{3}(1+\mathrm{Tr}\,R)^2-\frac{1}{4}K_i K_i\}^{1/3}} \tag{54}$$

where $R_{ij} = (-4f_\pi^{-2})S_{ji}^{\mathrm{LR}}$ and $K_i = d_{ijk}R_{jk}+\mathrm{i}f_{ijk}R_{jk}$. But this step has obviously solved the problem, as we now have a direct relationship between the Schwinger term and the independent components of U. Thus, where we have functions of R, $\mathscr{I}$, q_i and s_i forming the $(m, \bar{m})$ representation we can write them explicitly as functions of S_{ij}^{LR} by using equation (54), and therefore any mode of symmetry breaking can be written in the current–current model as a function of the Schwinger terms.

In conclusion, we have shown that using nonlinearly transforming meson fields it is possible to form the linear $(m, \bar{m})$ representations of chiral SU(3) × SU(3) as functions of the meson fields in a general coordinate system. The method used to complete this task was shown to be applicable to both SU(2) × SU(2) and SU(3) × SU(3) and extension to SU(n) × SU(n) is possible since the general formalism for constructing nonlinear realizations of the higher groups has been given in Barnes *et al* (1972b). The basic problem then is due to an increase in the number of Casimir invariants which is $n-1$ for SU(n) and therefore leads to $n-1$ partial differential equations. We hope to say more on this topic in a later paper.

The construction of symmetry breaking phenomenological Lagrangians is an obvious example of the use of the solutions that we have obtained and allows the full Lagrangian for physical processes to be written in any coordinate system. Further, we have established that Schwinger terms do provide a suitable mechanism for symmetry breaking in current theories, whatever $(m, \bar{m})+(\bar{m}, m)$ representations of SU(3) × SU(3) we require the symmetry breaking to be from, if they obey a further condition suggested by nonlinear models. The particular model described by Barnes and Isham (1970a, b) can therefore be reconstructed with $(3, \bar{3})+(\bar{3}, 3)$ symmetry breaking, although it is in the form of a complicated nonlinear function of the Schwinger terms. The physics of the model is still provided by going to a phenomenological Lagrangian, so that the outstanding problems of current–current theories still remain and clearly require further investigation.

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